

2018

Full Marks - 80

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions*The symbols used have their usual meaning*

1. a) Find the radius of curvature at any point  $(r, \theta)$  on the cardioid  $r = a(1 + \cos\theta)$  and find the area bounded by the parabola  $y^2 = 4ax$  and its latus rectum. 10

- b) Find the asymptotes to the curve  $xy - x + 2 = 0$ . 3

- c) Find the curvature of the curve  $y = \ln x$  at  $(e, 1)$ . 3

OR

- d) Trace the curve  $3ay^2 = x^2(x - a)$  and find the area of the enclosed between one arch of the cycloid  $x = a(\theta - \sin\theta)$ ,  $y = a(1 - \cos\theta)$  and its base. 10

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- e) Find the length of the arc of the curve  
 $y = 4 - x^2$  from  $(-2, 0)$  to  $(2, 0)$ . 3
- f) Find the volume of the sphere obtained by revolving the circle  $x^2 + y^2 = a^2$  about x-axis. 3
2. a) Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 - 4 = 0 = x^2 + y^2 + z^2 - 2x - 2y - 4$ , whose centre lies on the plane  $x + y + z = 1$  and find the equation of the cone whose vertex is  $(1, 1, 0)$  and guiding curve is  
 $x^2 + z^2 = 4, y = 0$ . 10
- b) Find the centre and radius of the sphere  
 $x^2 + y^2 + z^2 + 2x - 2y + 4z - 3 = 0$ . 3
- c) Identify the surface and its intersection with principal planes  $36x^2 + 9y^2 + 4z^2 = 36$ . 3

OR

[ 7 ]

- d) Solve  $x^2y'' + 2xy' - 20y = (x + 1)^2$ .  
and solve  $y'' + 2y' + 2y = 2, y(0) = 0, y'(0) = 1$   
using Laplace Transform. 10
- e) Find the Laplace Transform of  $e^{-2t}\cos t$ . 3
- f) Solve  $2y'' - 4y' + 8y = 0$ . 3

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e) Solve  $y = px + \frac{a}{p}$  3

f) Test the equation

$$(3x^2y + 8xy^2)dx + (x^3 + 8x^2y + 12y^2)dy = 0$$

for exactness. 3

5. a) Solve  $(D^2 - 5D + 6)y = 2e^x$

and solve  $(x^3D^3 + 2x^2D^2 - xD + 1)y = x^2 \log x$ . 10

b) Find the particular integral of

$$(D^2 + 4)y = 2 \tan 2x. \quad 3$$

c) Find the inverse transform of

$$\frac{3p - 2}{p^2 + 1}. \quad 3$$

OR

d) Find the equation of the enveloping cylinder of the sphere  $x^2 + y^2 + z^2 - 2x + 4y = 1$  whose generators are parallel to a line equally inclined to the coordinate axes and prove that the plane  $6x + 3y - 2z = 6$  touches the conicoid

Find the point of contact

$$36x^2 + 9y^2 - 4z^2 = 36. \quad 10$$

e) Show that  $x + y + z - 3$  touches the sphere

$$x^2 + y^2 + z^2 - x - y - z = 0. \quad 3$$

f) Identify the surface

$$36x^2 + 9y^2 - 4z^2 = -36. \quad 3$$

3. a) If  $u = f(y + ax) + g(y - ax)$ , prove that

$$\frac{\partial^2 x}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial y^2}$$

and if  $u = \sin^{-1} \frac{x^2 + y^2}{x + y}$ , show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u. \quad 10$$

[ 4 ]

b)  $f(x, y) = \frac{x^2}{x^2 + y^2}, (x, y) \neq (0, 0)$   
 $= 0, (x, y) = (0, 0)$

Show that the repeated limit does not exist when  
 $(x, y) \rightarrow (0, 0)$ . 3

c)  $f(x, y) = x \sin^{-1} y + y \sin^{-1} x$ .  
 find  $\frac{\partial^2 u}{\partial x^2}$  and  $\frac{\partial^2 u}{\partial y^2}$ . 3

OR

d) Test the function  $x^2 - xy + y^4$  for relative maxima and relative minima and  
 if  $u = (f(y - z, z - x, x - y))$ , prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \quad 10$$

e) If  $x = r \cos \theta, y = r \sin \theta$ ,  
 then find  $\frac{\partial(x, y)}{\partial(r, \theta)}$  3

[ 5 ]

f) Show that the function  $|x| + |y|$  is not continuous at origin. 3

4. a) Solve  $2xy^3 + 3x^2y^2 \frac{dy}{dx} = 0$   
 and solve  $(y - px)(p - 1) = p$ . 10

b) Solve:  
 $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$ . 3

c) Find the integrating factor of  
 $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$ . 3

OR

d) Solve  $\frac{dy}{dx} = \frac{x + y + 4}{x - y - 6}$   
 and solve  $y^2 + xyp - x^2p^2 = 0$ . 10