2017

Full Marks - 80

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer ***all*** questions selecting either {(a),(b),(c)} or {(d),(e),(f)} from each question

The symbols used have their usual meaning

1. a) Prove that the radius of curvature for cartesian curves y = f(x) is

$$\rho = \frac{\left(1+y_1^2\right)^{3/2}}{y_2},$$

where $y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$ 5

b) Find the area of the loop of the curve

$$3\,ay^2 = x(x-a)^2.$$ 5

c) i) Find out the radius of curvature for the pedal curve $p^2 = ar$ 3

ii) Find the asymptotes of the curve xy

$xy + 2x + 5 = 0$. 3

OR

d) Trace the curve $y^2 = x^3$. 5

e) Find the volume of the solid generated by roating the area enclosed between $y^2 = x^3 + 5x$ and the lines $x = 2$ and $x = 4$ about x-axis. 5

f) i) Find the length of that arc of the curve $y = x(2 - x)$ from $x = 0$ to $x = 2$. 3

ii) Find the area of the curved surface of a hemisphere of radius a. 3

2. a) Prove that the plane $2x - 2y + z + 16 = 0$ touches the sphere $x^2 + y^2 + z^2 + 2x - 4y + 2z - 3 = 0$ and find also the point of contact. 5

b) Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z - 1 = 0$ with the vertex at (1, 1, 1). 5

e) Use the Laplace transformation to solve the initial value problem

$y'' - 2y' - 3y = 0, y(0) = 1, y'(0) = 7$. 5

f) i) Find the Laplace transform of $t^2 \cos t$. 3

ii) Find the general solution of the differential equation

$x^2y'' - 3xy' + 3y = 0$. 3

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e) Solve $y = 2px + y^2p^3$ 5

f) i) Test the equation

$$xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$$

for exactness. 3

ii) Solve $y^2 + p^2 = a^2$. 3

5. a) Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$. 5

b) Solve $(x^2D^2 - 3xD + 4)y = 2x^2$. 5

c) i) Find particular solution of the equation

$(D^2 + 1)y = \sec x$. 3

ii) Find the inverse transform of

$$\frac{2p+1}{p^2-4}.$$ 3

OR

d) Solve $xy'' - (2x - 1)y' + (x - 1)y = 0$. 5

c) i) Find the equation of the sphere whose centre is (1, 0, –1) and which passes through the point (2, –1, 1). 3

ii) Prove that the origin is the centre of the ellipsoid. 3

OR

d) Find the equation of the right circular cone with vertex (1, –2, –1), semi vertical angle $\frac{\pi}{3}$ and the axis $\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z+1}{5}$ 5

e) Prove that the plane $24x + 51y - 28z + 7 = 0$ touches the concoid $12x^2 - 17y^2 + 7z^2 = 7$. Find the point of contact. 5

f) i) Name the surfaces

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, -\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$ 3

ii) Define the following Normal of the conicoid, Right circular cone. 3

[Turn Over

3. a) If $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & , x \neq 0, y \neq 0 \\ 0 & , x = 0, y = 0 \end{cases}$

then show that both the partial derivatives at (0, 0) exist but the function is not continuous at (0, 0). 5

b) If $z = \sec^{-1} \dfrac{x^3 - y^3}{x + y}$,

then show that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 2\cot z$. 5

c) i) Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2 + y^4}$ 3

ii) If $(\tan x)^y + y^{\cot x} = a$, find $\dfrac{dy}{dx}$. 3

OR

d) Show that for $0 < \theta < 1$,

$e^{ax} \sin by = by + abxy + e^{a\theta x}\left[\left(a^3x^3 - 3ab^2xy^2\right)\right.$
$\left.\sin b\theta y + 3a^2bx^2y - b^3y^3\right) \cos\theta y$. 5

e) Find the maxima and minima of the function

$f(x, y) = x^3 + y^3 - 3x - 12y + 20$. 5

f) i) Prove that $\int_0^1 dx \int_0^1 \dfrac{x - y}{(x + y)^3} dy = \dfrac{1}{2}$. 3

ii) If $f(x,y) = \dfrac{xy}{x + y}$,

find $f_x(x, y)$ at (2, 1) and $f_y(x, y)$ at (2, 1) 3

4. a) Solve $\dfrac{dy}{dx} + xy = x^3y^3$ 5

b) Solve $y = x + p^2 - \dfrac{2}{3}p^3$ 5

c) i) Find the integrating factor of the equation

$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$. 3

ii) Solve $p^2 - 5p + 6 = 0$. 3

OR

d) Solve $\dfrac{dy}{dx} = \dfrac{y + x + 1}{2x + 2y + 1}$ 5